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Research Note — Lenses

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1. Thesis

The textbook lens is a four-fold idealisation: thin, paraxial, monochromatic, perfectly spherical. Module 03 (single slit) showed that the point-source assumption fails already within a single slit; Module 04 (camera model) touched on thin vs. thick lens. Module 05 examines the lensmaker formula $1/f = (n-1)(1/R_1 - 1/R_2)$ along each of the four axes separately and shows: **the ideal lens exists only in the paraxial monochromatic limit — once any of the four idealisations is dropped, the real focal point deviates measurably.** This note closes a central gap in the optics-lab strand.

2. Assumptions

1. Scalar paraxial ray optics (no diffraction, no vector character, small angles).
2. Lens made of homogeneous glass, two spherical surfaces with radii R_1 and R_2.
3. Refractive index n constant across the lens (homogeneous).
4. Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$ holds exactly.
5. Lens surrounded by air ($n_{\text{air}} = 1$).
6. Light incident parallel to optical axis (collimated).
7. Circular aperture, radius equal to half the physical lens opening.

3. Mathematical model

M1 — ideal thin lens (textbook):

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

Properties: All rays converge to one focal point; all wavelengths converge to one focal point; principal planes coincide with the lens vertex.

M2 — thick lens (paraxial with thickness correction):

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{n R_1 R_2}\right).$$

M3 — spherical aberration: exact raytracing through two spherical surfaces using vectorial Snell's law. Ray height h as parameter; $f(h)$ instead of single f. Marginal rays focus at $f(h) < f(0)$ — the spherical lens “over-focuses” marginal rays.

M4 — chromatic aberration: $n = n(\lambda)$ via Cauchy formula:

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}.$$

For BK7 glass: $A = 1.5046$, $B = 4200$ nm², $C = 2.85 \cdot 10^8$ nm⁴.

Symbol	Meaning	Unit
R_1, R_2	radii of curvature (convention: $R > 0$ convex toward incident light)	mm
n	refractive index	—
d	centre thickness of lens	mm
f	focal length	mm
h	ray height above optical axis	mm
λ	wavelength	nm

4. Parameters

Four configurations along the four axes:

Scenario	R_1 / mm	R_2 / mm	n	d / mm	Aperture / mm	
A thin	$+100$	-100	1.5	2	15	tex lim
B thick	$+50$	-50	1.5	15	15	thi co
C strong	$+30$	-30	1.5	20	12	str spl ab
D chromatic	$+50$	-50	$n_{\text{BK7}}(\lambda)$	10	10	F/c wa

Tolerance set in advance: $\varepsilon = 1\%$ relative focal-length deviation. Rationale: 1% is the standard workshop tolerance; relative focal length is the quantity that matters directly in practice (photo, microscope, glasses).

5. Prediction

Scenario	Expectation
A thin	M1 should be supported (very thin, $d/R \sim 0.02$)
B thick	M1 refuted via thickness correction
C strong	M1 strongly refuted via spherical aberration and thickness
D chromatic	$\Delta f / f \sim 2$–3% between blue and red for BK7

6. Falsification criterion

Four separate criteria:

- **M1 refuted via thickness** if $|f_{\text{thick}} - f_{\text{thin}}|/f_{\text{thin}} > \varepsilon$.
- **M1 refuted via spherical aberration** if $|f(h=0.9 \cdot \text{aperture}) - f(h \approx 0)|/f(h \approx 0) > \varepsilon$.
- **M1 refuted via chromatic aberration** if $|f(\lambda_{\text{F}}) - f(\lambda_{\text{C}})|/f(\lambda_{\text{d}}) > \varepsilon$.
- M1 is refuted overall if **any** of the three criteria applies.

7. Competing models

- **K1 — thin paraxial lens (M1).**
- **K2 — thick paraxial lens (M2).**
- **K3 — exact raytracing (M3):** vectorial Snell at both spherical surfaces — tested model.
- **K4 — wavelength-dependent lens (M4).**
- **K5 — Achromat:** doublet of two glasses (crown + flint) for correction of chromatic aberration at two wavelengths.
- **K6 — Aspheric lens:** $R(h)$ as function of height, corrects spherical aberration.
- **K7 — Fresnel lens:** thin via concentric grooves — own class, future module.
- **K8 — Diffraction-limited aperture:** Module 03 — diffraction as fundamental resolution limit, independent of aberrations.

8. Domain of failure

- **Very strong curvature** ($h \sim R$): paraxial theory entirely invalid.
- **Refractive index $n \to 1$**: lens ineffective.
- **Refractive index $n \to \infty$**: total internal reflection at exit surface.
- **Very small aperture** ($h \to 0$): paraxial approximation exact, but diffraction limit (Module 03) dominates.
- **Off-axis light**: coma and astigmatism, separate aberration classes.
- **Inhomogeneous glass**: schlieren, GRIN lenses — separate topic.

9. Simulation

Numerical evaluation in `linsen_simulation.py`:

1. M1: direct evaluation of lensmaker formula.
2. M2: extended lensmaker formula with thickness correction.
3. M3: exact vectorial raytracer through two spherical surfaces. Per ray: entry point on first sphere, Snell refraction, propagation through glass, exit on second sphere, second Snell refraction, intersection with optical axis. Ray heights $h = 0.1$ mm (paraxial) and $h = 0.9 \cdot$ aperture (marginal).
4. M4 (scenario D): Cauchy formula for BK7 at three standard wavelengths (Fraunhofer F 486.1 nm, d 587.6 nm, C 656.3 nm) plus dense wavelength curve across the visible spectrum.

Results in `linsen_results.json`. Plots as `linsen_A_duenn.svg`, `linsen_B_dick.svg`, `linsen_C_stark.svg`, `linsen_D_chromatisch.svg`.

10. Result

10.1 A thin lens (\(R = \pm 100\) mm, \(d = 2\) mm, \(n = 1.5\))

Quantity	Value
\(f\) thin (lensmaker)	\(100.00\) mm
\(f\) thick (correction)	\(100.33\) mm
\(f\) paraxial raytracing (\(h = 0.1\) mm)	\(99.67\) mm
\(f\) marginal (\(h = 13.5\) mm = 90% aperture)	\(96.59\) mm
Thickness deviation	\(0.33\%\) → supported
Aberration deviation	\(3.1\%\) → **refuted**

10.2 B thick biconvex (\(R = \pm 50\) mm, \(d = 15\) mm, \(n = 1.5\))

Quantity	Value
\(f\) thin	\(50.00\) mm
\(f\) thick	\(52.63\) mm
\(f\) paraxial raytracing	\(47.37\) mm
\(f\) marginal	\(41.89\) mm
Thickness deviation	\(5.3\%\) → **refuted**
Aberration deviation	\(11.6\%\) → **refuted**

10.3 C strong biconvex (\(R = \pm 30\) mm, \(d = 20\) mm)

Quantity	Value
\(f\) thin	\(30.00\) mm
\(f\) thick	\(33.75\) mm
\(f\) paraxial raytracing	\(26.25\) mm
\(f\) marginal	\(21.53\) mm
Thickness deviation	\(12.5\%\) → **refuted**
Aberration deviation	\(18.0\%\) → **refuted**

10.4 D chromatic (BK7 biconvex \(R = \pm 50\) mm, \(d = 10\) mm)

Wavelength	\(n_{\text{BK7}}\)	\(f\) raytracing
Fraunhofer F, blue, 486 nm	\(1.5275\)	\(45.70\) mm
Fraunhofer d, green, 588 nm	\(1.5192\)	\(46.45\) mm
Fraunhofer C, red, 656 nm	\(1.5159\)	\(46.75\) mm
Longitudinal chromatic aberration (red − blue)	—	\(1.05\) mm
Relative blue-red deviation	—	\(2.3\%\) → **refuted**

Plots: `linsen_A_duenn.svg`, `linsen_B_dick.svg`, `linsen_C_stark.svg`, `linsen_D_chromatisch.svg`.

11. Assessment

Even the thin textbook lens (A) is refuted — via spherical aberration, not via the thickness correction. This is the most important methodological lesson of this note: the “ideal thin lens” exists only paraxially. As soon as one considers rays off the axis (at just 13.5 mm ray height, corresponding to f/3.7 half-aperture), the real focal point deviates by 3%. In practice this means: every photo lens with large aperture and spherical surfaces shows spherical aberration; this is why premium objectives use aspheric lens elements.

B and C show the transition to practical aberrations — for a 15 mm thick lens with $R = 50$ mm, the thickness correction is 5%, spherical aberration is 12%. At very strong curvature (C, $R = 30$ mm), the focal-length difference between axis and rim reaches 18%. This matches the experience that short-focal-length lenses (“wide-angle”) are particularly demanding to compute and unusable without correction.

Chromatic aberration (D) shows $\Delta f = 1.05$ mm between Fraunhofer-F (blue, 486 nm) and Fraunhofer-C (red, 656 nm) for a 47 mm lens — that is 2.3%. Visible as coloured fringes at high-contrast edges. The historically decisive step was the achromat construction (Dollond 1758, later Fraunhofer): a doublet of crown glass (BK7, low dispersion) and flint glass (SF6, high dispersion) which forces the Δf between two wavelengths to exactly zero. Apochromats correct three wavelengths.

Joint assessment: Four different falsification axes, four different historical solutions — thin-lens assumption solved by principal-plane treatment, spherical aberration by asphericity or assemblies of small lenses, chromatic aberration by achromat doublet, paraxial assumption by full raytracing (today the software standard in lens design). Today’s optical industry is essentially the systematic removal of the four idealisations that are carried unquestioned through school physics.

Connection to predecessor modules: Module 03 showed that every finite aperture produces diffraction — a **fundamental** resolution limit, not a correctable design flaw. Even a fully aberration-corrected lens produces an Airy-disc focal spot of size $\sim 1.22 \lambda f/D$. The aberrations from this note and the diffraction limit from Module 03 are complementary: aberration is geometric-optical (rays do not meet), diffraction is wave-optical (even perfect rays form an extended spot).

12. Next test

Module 06 Prisms. The prism generalises chromatic aberration into a usable tool: by suitable choice of glass type and apex angle, light is systematically spectrally decomposed. Prediction: minimum deviation $\delta_{\min} = 2\arcsin(n\sin(\alpha/2)) - \alpha$ for symmetric ray path; wavelength dispersion linear to quadratic in the visible range. Test against the four-line spectrum of a prism (Fraunhofer lines as calibration marks).

Additionally: **Module 07 Light sources** (coherent vs. incoherent, line vs. continuum spectra) as complement to the optics family.

Appendix A — Cross-references

- Practice explanation: `PRAXIS-Linsen-EN.md`
- Companion paper: `BEGLEITPAPIER-Linsen-EN.md`
- Tool: `PERSPECTIVA.html` — named lens tool, *lat. perspectiva*, medieval science of vision
- Code: `linsen_simulation.py`
- Plots: `linsen_A_duenn.svg`, `linsen_B_dick.svg`, `linsen_C_stark.svg`, `linsen_D_chromatisch.svg`
- Result data: `linsen_results.json`
- Predecessor modules: Module 03 Single Slit (diffraction limit), Module 04 Camera Model (thin vs. thick).
- Related modules: Module 06 Prisms (forthcoming), Module 07 Light Sources (forthcoming), Module 11 Shadow Theatre (geometric optics as limit).

Appendix B — External stocktaking

- **Hecht (2017)** *Optics*, 5th ed., ch. 5–6: paraxial optics, lenses, aberrations.
- **Born & Wolf (1999)** *Principles of Optics*, 7th ed., ch. 4: complete treatment of ray optics and aberrations.
- **Mahajan (1998)** *Optical Imaging and Aberrations*: Seidel aberrations, aspheric correction.
- **Smith (2008)** *Modern Optical Engineering*, 4th ed.: industrial practice.
- **PhET** "Geometric Optics" — didactic lens simulation, paraxial only.
- **OpticsBench, OSLO, Zemax** — professional raytracers.

What the workshop note adds: four separate falsification axes in a consistent format, own vectorial raytracer, explicit bridge to Module 03 (diffraction limit vs. aberrations as complementary resolution limits), bridge to Module 06 Prisms.

Appendix C — Sources

- Hecht, E. (2017). *Optics*, 5th ed. Pearson.
- Born, M.; Wolf, E. (1999). *Principles of Optics*, 7th ed. Cambridge.
- Mahajan, V. N. (1998). *Optical Imaging and Aberrations*. SPIE Press.
- Smith, W. J. (2008). *Modern Optical Engineering*, 4th ed. McGraw-Hill.
- Schott AG (ongoing). *Optical Glass Data Sheets* (BK7, F2, SF6 etc.).
- Dollond, J. (1758). On the different refrangibility of light. *Phil. Trans.* 50:733.

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